LIBERTY PAPER SET

STD. 12 : Mathematics

Full Solution

Time : 3 Hours **ASSIGNMENT PAPER 7**

PART A

1. (D) **2.** (C) **3.** (D) **4.** (C) **5.** (A) **6.** (C) **7.** (B) **8.** (A) **9.** (C) **10.** (D) **11.** (A) **12.** (D) **13.** (D) **14.** (A) **15.** (B) **16.** (A) **17.** (B) **18.** (B) **19.** (A) **20.** (C) **21.** (B) **22.** (B) **23.** (D) **24.** (D) **25.** (A) **26.** (D) **27.** (B) **28.** (C) **29.** (C) **30.** (A) **31.** (B) **32.** (B) **33.** (D) **34.** (C) **35.** (D) **36.** (D) **37.** (A) **38.** (B) **39.** (D) **40.** (C) **41.** (B) **42.** (A) **43.** (C) **44.** (A) **45.** (C) **46.** (B) **47.** (B) **48.** (C) **49.** (A) **50.** (A)

PART B

SECTION A

1.

$\tan^{-1}\left(\dfrac{\cos x - \sin x}{\cos x + \sin x}\right)$

$= \tan^{-1}\left(\dfrac{1 - \tan x}{1 + \tan x}\right)$

$= \tan^{-1}\left(\dfrac{\tan\frac{\pi}{4} - \tan x}{1 + \tan\frac{\pi}{4}\tan x}\right)$

$= \tan^{-1}\left(\tan\left(\dfrac{\pi}{4} - x\right)\right)$

Here, $-\dfrac{\pi}{4} < x < \dfrac{3\pi}{4}$

$\Rightarrow -\dfrac{3\pi}{4} < -x < \dfrac{\pi}{4}$

$\Rightarrow -\dfrac{\pi}{2} < \dfrac{\pi}{4} - x < \dfrac{\pi}{2}$

$\Rightarrow \left(\dfrac{\pi}{4} - x\right) \in \left(-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right)$

$= \dfrac{\pi}{4} - x$

2.

$\sin\left(2\tan^{-1}\dfrac{2}{3}\right) + \cos\left(\tan^{-1}\sqrt{3}\right)$

$\sin\left(2\tan^{-1}\dfrac{2}{3}\right)$

Now, Take $\tan^{-1}\dfrac{2}{3} = \theta$,

$\therefore \tan\theta = \dfrac{2}{3}$

$\therefore \sin 2\theta = \dfrac{2\tan\theta}{1 + \tan^2\theta}$

$= \dfrac{2\left(\frac{2}{3}\right)}{1 + \frac{4}{9}}$

$= \dfrac{\frac{4}{3}}{\frac{13}{9}}$

$= \dfrac{12}{13}$

$= \sin\left(2\tan^{-1}\dfrac{2}{3}\right) + \cos\left(\tan^{-1}\sqrt{3}\right)$

$= \dfrac{12}{13} + \cos\left(\tan^{-1}\left(\tan\dfrac{\pi}{3}\right)\right)$

$= \dfrac{12}{13} + \cos\dfrac{\pi}{3} \quad \left(\because \dfrac{\pi}{3} \in \left(\dfrac{-\pi}{2}, \dfrac{\pi}{2}\right)\right)$

$= \dfrac{12}{13} + \dfrac{1}{2}$

$= \dfrac{24 + 13}{26}$

$= \dfrac{37}{26}$

3.

$x^y = e^{x-y}$

$\log x^y = \log e^{x-y}$

$\therefore y \log x = (x - y) \log e$

$\therefore y \log x = x - y$

$\therefore \quad y \log x + y = x$

$\therefore \quad y (\log x + 1) = x$

$\therefore \quad y = \dfrac{x}{1+\log x}$

Now, Differentiate w.r.t. x,

$$\therefore \quad \frac{dy}{dx} = \frac{d}{dx}\left(\frac{x}{1+\log x}\right)$$

$$= \frac{(1+\log x)\frac{d}{dx}(x) - x\frac{d}{dx}(1+\log x)}{(1+\log x)^2}$$

$$= \frac{(1+\log x)(1) - x\cdot\left(0+\frac{1}{x}\right)}{(1+\log x)^2}$$

$$= \frac{1+\log x - 1}{(1+\log x)^2}$$

$$\therefore \quad \frac{dy}{dx} = \frac{\log x}{(1+\log x)^2}$$

4.

$$\text{I} = \int \frac{dx}{\sqrt{(x-1)(x-2)}}$$

$$= \int \frac{dx}{\sqrt{x^2 - 3x + 2}}$$

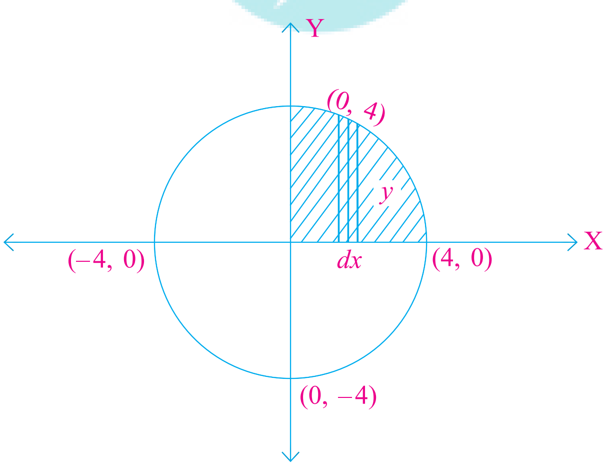
$$= \int \frac{dx}{\sqrt{x^2 - 2\left(\frac{3}{2}x\right) + \frac{9}{4} - \frac{9}{4} + 2}}$$

$$\therefore \quad \text{I} = \int \frac{dx}{\sqrt{\left(x+\frac{3}{2}\right)^2 - \frac{1}{4}}}$$

$$= \int \frac{dx}{\sqrt{\left(x+\frac{3}{2}\right)^2 - \left(\frac{1}{2}\right)^2}}$$

$$\therefore \quad \text{I} = \log\left|x + \frac{3}{2} + \sqrt{x^2 - 3x + 2}\right| + c$$

5.



$x^2 + y^2 = 16$

$\therefore \quad y^2 = 16 - x^2$

$\therefore \quad y = \sqrt{16 - x^2}$

$$\rightarrow \quad \text{I} = \int_0^4 y\, dx$$

$$= \int_0^4 16 - x^2 \; dx$$

$$= \int_0^4 \sqrt{4^2 - x^2} \; dx$$

$$= \left[\frac{x}{2}\sqrt{4^2 - x^2} + \frac{16}{2}\sin^{-1}\left(\frac{x}{4}\right)\right]_0^4$$

$= 8 \sin^{-1}(1) - 0$

$= 8\left(\dfrac{\pi}{2}\right)$

$= 4\pi$

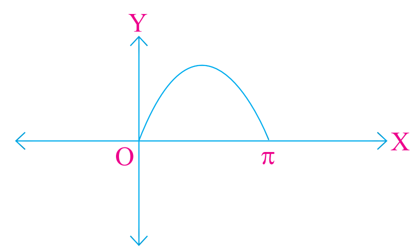
$\therefore$ Required area $= 4|\text{I}|$

$= 4(4\pi)$

$= 16\pi$ sq. unit

6.

$y = \sin x$, $x = 0$ and $x = \pi$ from this area of region is A.



$$\text{I} = \int_0^{\pi} \sin x\, dx$$

$= (-\cos x)_0^{\pi}$

$= -\cos \pi + \cos 0$

$= -(-1) + 1$

$= 1 + 1$

$= 2$

$\therefore \quad \text{A} = |\text{I}|$

$= 2$ sq. unit

7.

$$\frac{dy}{dx} + \sqrt{\frac{1-y^2}{1-x^2}} = 0$$

$$\therefore \quad \frac{dy}{dx} = -\sqrt{\frac{1-y^2}{1-x^2}}$$

$$\therefore \quad \frac{dy}{dx} = -\frac{\sqrt{1-y^2}}{\sqrt{1-x^2}}$$

$$\therefore \quad \frac{dy}{\sqrt{1-y^2}} = \frac{-dx}{\sqrt{1-x^2}}$$

→ Integrate both the sides,

$$\therefore \quad \int \frac{dy}{\sqrt{1-y^2}} = \int \frac{-dx}{\sqrt{1-x^2}}$$

$\therefore\ sin^{-1}(y) = -sin^{-1}(x) + c$

$\therefore\ sin^{-1}(x) + sin^{-1}(y) = c$

Which is required general solution.

8.

A(1, 1, 2), B(2, 3, 5), C(1, 5, 5)

$\overrightarrow{AB} = (2, 3, 5) - (1, 1, 2)$

$= (1, 2, 3)$

$= \hat{i} + 2\hat{j} + 3\hat{k}$

$\overrightarrow{BC} = (1, 5, 5) - (2, 3, 5)$

$= (-1, 2, 0)$

$= -\hat{i} + 2\hat{j} + 0\hat{k}$

Area of $\Delta ABC = \frac{1}{2}|\overrightarrow{AB} \times \overrightarrow{BC}|$ .... (1)

$$\text{Now, } \overrightarrow{AB} \times \overrightarrow{BC} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 3 \\ -1 & 2 & 0 \end{vmatrix}$$

$= -6\hat{i} - 3\hat{j} + 4\hat{k}$

$|\overrightarrow{AB} \times \overrightarrow{BC}| = \sqrt{36+9+16}$

$= \sqrt{61}$

From equation (1), $\Delta = \frac{\sqrt{61}}{2}$ sq. unit

9.

$\vec{b_1} = 3\hat{i} + 2\hat{j} + 6\hat{k}$

$\vec{b_2} = \hat{i} + 2\hat{j} + 2\hat{k}$

Suppose, Angle between two lines is α then,

$$cos\ \alpha = \frac{|\vec{b_1} \cdot \vec{b_2}|}{|\vec{b_1}||\vec{b_2}|} \quad \text{......... (1)}$$

$\vec{b_1} \cdot \vec{b_2} = (3\hat{i} + 2\hat{j} + 6\hat{k}) \cdot (\hat{i} + 2\hat{j} + 2\hat{k})$

$= 3 + 4 + 12$

$= 19$

$|\vec{b_1}| = \sqrt{9+4+36}$

$= \sqrt{49}$

$= 7$

$|\vec{b_2}| = \sqrt{1+4+4}$

$= \sqrt{9}$

$= 3$

From equation (1),

$$cos\ \alpha = \frac{|19|}{(7)(3)}$$

$$= \frac{19}{21}$$

$$\therefore \quad \alpha = cos^{-1}\left(\frac{19}{21}\right)$$

Therefore, Angle between two line is $cos^{-1}\left(\frac{19}{21}\right)$.

10.

$\vec{r} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(\hat{i} - 3\hat{j} + 2\hat{k}), \lambda \in R$

$\vec{a_1} = \hat{i} + 2\hat{j} + 3\hat{k};$

$\vec{b_1} = \hat{i} - 3\hat{j} + 2\hat{k}$

and $\vec{r} = 4\hat{i} + 5\hat{j} + 6\hat{k}$

$+ \mu(2\hat{i} + 3\hat{j} + \hat{k}); \mu \in R$

$\vec{a_2} = 4\hat{i} + 5\hat{j} + 6\hat{k};$

$\vec{b_2} = 2\hat{i} + 3\hat{j} + \hat{k}$

$$\text{Now, } \vec{b_1} \times \vec{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -3 & 2 \\ 2 & 3 & 1 \end{vmatrix}$$

$= -9\hat{i} + 3\hat{j} + 9\hat{k}$

$\neq \vec{0}$

Lines are intersecting lines or skew lines

$\vec{a_2} - \vec{a_1} = 3\hat{i} + 3\hat{j} + 3\hat{k}$

$|\vec{b_1} \times \vec{b_2}| = \sqrt{81+9+81}$

$= \sqrt{171}$

Now, $(\vec{a_2} - \vec{a_1}) \cdot (\vec{b_1} \times \vec{b_2})$

$= (3\hat{i} + 3\hat{j} + 3\hat{k}) \cdot (-9\hat{i} + 3\hat{j} + 9\hat{k})$

$= -27 + 9 + 27$

$= 9$

$\neq 0$

$\therefore$ Line are skew lines.

Shortest distance between two skew lines,

$$= \frac{|(\vec{a_2} - \vec{a_1}) \cdot (\vec{b_1} \times \vec{b_2})|}{|\vec{b_1} \times \vec{b_2}|}$$

$$= \frac{9}{\sqrt{171}}$$

$$= \frac{9}{3\sqrt{19}}$$

$$= \frac{3}{\sqrt{19}} \text{ unit}$$

11.

If a multiple of 3 comes up, throw the die again and if any other number comes, toss a coin,

Die is thrown

3, 6 again throw die	1, 2, 4, 5 coin is tossed
↓	↓
1, 2, 3, 4, 5, 6	H, T

S = {(3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6), (6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6), (1, H), (1, T), (2, H), (2, T), (4, H), (4, T), (5, H), (5, T)}

Event A : at least one die shows a 3,

A = {(3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6), (6, 3)}

$\therefore \quad r = 7$

$\therefore \quad P(A) = \dfrac{7}{20}$

Event B : tail on coin

B = {(1, T), (2, T), (4, T), (5, T)}

$\therefore \quad r = 4$

$\therefore \quad P(B) = \dfrac{4}{20}$

$\therefore \quad A \cap B = \phi$

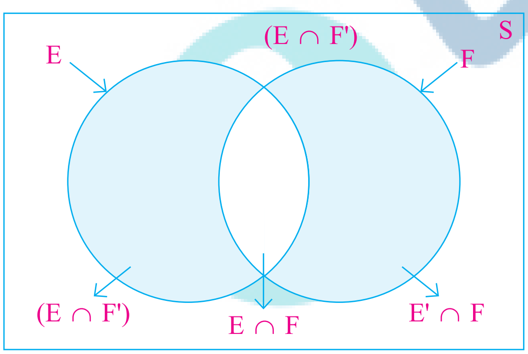
$\therefore \quad P(A \cap B) = 0$

$\therefore \quad P(B \mid A) = 0$

12.

Since E and F are independent, we have,

$P(E \cap F) = P(E) \cdot P(F)$ .... (1)



From the venn diagram in Fig., it is clear that $E \cap F$ and $E \cap F'$ are mutually exclusive events and also $E = (E \cap F) \cup (E \cap F')$.

Therefore $P(E) = (E \cap F) + P(E \cap F')$

or $P(E \cap F') = P(E) - P(E \cap F)$

$= P(E) - P(E)\,P(F)$ (by (1))

$= P(E)\,(1 - P(F))$

$= P(E) \cdot P(F')$

Hence, E and F' are independent.

SECTION B

13.

Here $f : R \to R$, $f(x) = \begin{cases} 1, & x > 0 \\ 0; & x = 0 \\ -1; & x < 0 \end{cases}$

Take, $x_1 = 2$ and $x_2 = 3$,

$f(x_1) = 1;\ f(x_2) = 1$

$x_1 \neq x_2$ But $f(x_1) = f(x_2)$

$\therefore$ f is not one-one function.

(Here, Take infinite value such that x_1 and x_2 are different.)

Range of signum function = {–1, 0, 1} ≠ co-domain

$\therefore$ f is not onto function.

Note : Here $f : R \to R$

$f(x) = \begin{cases} \dfrac{|x|}{x}, & x \neq 0 \\ 0 ; & x = 0 \end{cases}$ is also signum function.

14.

Take, $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$,

$B = \begin{bmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{bmatrix}$

Suppose, order of matrix X is $m \times n$

Here, order of matrix

A is 2×3 and order of matrix

B is 2×3.

For defining XA,

Number of column of X

= Number of Row of A.

Find the matrix X

So that $X\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} = \begin{bmatrix} -7 & -8 & -9 \\ 2 & 4 & 4 \end{bmatrix}$

$\therefore \quad n = 2$

Order of XA = order of B

$\therefore \quad m \times 3 = 2 \times 3$

$\therefore \quad m = 2$

Therefore, order of matrix X must be taken as 2×2.

Suppose, $X = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

Here, XA = B

$\therefore \quad \begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} = \begin{bmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{bmatrix}$

$\therefore \quad \begin{bmatrix} a+4b & 2a+5b & 3a+6b \\ c+4d & 2c+5d & 3c+6d \end{bmatrix} = \begin{bmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{bmatrix}$

$\therefore \quad a + 4b = -7 \quad \ldots\ldots\ldots\ldots \text{(i)}$

$2a + 5b = -8 \quad \ldots\ldots\ldots\ldots \text{(ii)}$

$c + 4d = 2 \quad \ldots\ldots\ldots\ldots \text{(iii)}$

$2c + 5d = 4 \quad \ldots\ldots\ldots\ldots \text{(iv)}$

Solving equation (1) and (2),

$$2a + 8b = -14$$
$$2a + 5b = -8$$
$$- \quad - \quad +$$
$$3b = -6$$
$$b = -2$$

Put $b = -2$ in equation (1),

$$2a - 16 = -14$$
$$\therefore \quad 2a = 2$$
$$\therefore \quad a = 1$$

Solving equation (iii) and (iv),

$$2c + 8d = 4$$
$$2c + 5d = 4$$
$$- \quad - \quad -$$
$$3d = 0$$
$$\therefore \quad d = 0$$

Put $d = 0$ in equation (iii),

$$2c + 0 = 4$$
$$\therefore c = 2$$

Therefore, $a = 1,\ b = -2,\ c = 2,\ d = 0$

$$\therefore \quad X = \begin{bmatrix} 1 & -2 \\ 2 & 0 \end{bmatrix}$$

15.

$$\Delta = \begin{vmatrix} 5 & 3 & 8 \\ 2 & 0 & 1 \\ 1 & 2 & 3 \end{vmatrix}$$

Co-factor of the element 2 $\quad A_{21} = (-1)^3 \begin{vmatrix} 3 & 8 \\ 2 & 3 \end{vmatrix}$

$= (-1)\,[9 - 16]$

$= (-1)\,(-7)$

$= 7$

Co-factor of the element 0 $\quad A_{22} = (-1)^4 \begin{vmatrix} 5 & 8 \\ 1 & 3 \end{vmatrix}$

$= (1)\,[15 - 8]$

$= 7$

Co-factor of the element 1 $\quad A_{23} = (-1)^5 \begin{vmatrix} 5 & 3 \\ 1 & 2 \end{vmatrix}$

$= (-1)\,[10 - 3]$

$= (-1)\,(7)$

$= -7$

$$\Delta = a_{21}A_{21} + a_{22}A_{22} + a_{23}A_{23}$$
$$= (2)(7) + (0)(7) + (1)(-7)$$
$$= 14 + 0 - 7$$
$$= 7$$
$$\Delta = 7$$

16.

Take both the sides *log*,

$$y\log(\cos x) = x\log(\cos y)$$

$$\therefore\ y\frac{d}{dx}\log(\cos x) + \log(\cos x)\frac{d}{dx}y = x\frac{d}{dx}\log(\cos y) + \log(\cos y)\frac{d}{dx}x$$

$$\therefore\ y \cdot \frac{1}{\cos x}(-\sin x) + \log(\cos x)\frac{dy}{dx} = x\frac{1}{\cos y}(-\sin y)\frac{dy}{dx} + \log(\cos y)$$

$$\therefore\ -y\tan x + \log(\cos x)\frac{dy}{dx} = -x\tan y\frac{dy}{dx} + \log(\cos y)$$

$$\therefore\ \log(\cos x)\frac{dy}{dx} + x\tan y\frac{dy}{dx} = \log(\cos y) + y\tan x$$

$$\therefore\ \frac{dy}{dx}[\log(\cos x) + x\tan y] = \log(\cos y) + y\tan x$$

$$\therefore\ \frac{dy}{dx} = \frac{y\tan x + \log(\cos y)}{x\tan y + \log(\cos x)}$$

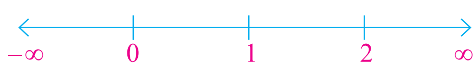
17.

$$f(x) = (x(x-2))^2$$
$$f(x) = (x(x-2))^2$$
$$= x^2(x-2)^2$$
$$= x^2(x^2 - 4x + 4)$$
$$f(x) = x^4 - 4x^3 + 4x^2$$
$$\therefore f'(x) = 4x^3 - 12x^2 + 8x$$
$$= 4x(x^2 - 3x + 2)$$
$$= 4x(x-2)(x-1)$$

$\rightarrow$ For finding intervals,

$$f'(x) = 0$$
$$\therefore \quad 4x(x-2)(x-1) = 0$$

$\therefore \quad x = 0 \quad | \quad x - 2 = 0,\ x = 2 \quad | \quad x - 1 = 0,\ x = 1$



$\rightarrow \forall\, x \in (-\infty, 0) \quad \Rightarrow x < 0,\ x - 2 < 0,\ x - 1 < 0$

$\Rightarrow x(x-2)(x-1) < 0$

$\Rightarrow 4x(x-2)(x-1) < 0$

$\Rightarrow f'(x) < 0$

$\therefore$ f is strictly decreasing function in the interval of $(-\infty, 0)$.

$\rightarrow \forall\, x \in (0, 1) \quad \Rightarrow x > 0,\ x - 2 < 0,\ x - 1 < 0$

$\Rightarrow x(x-2)(x-1) > 0$

$\Rightarrow 4x(x-2)(x-1) > 0$

$\Rightarrow f'(x) > 0$

$\therefore$ f is strictly increasing function in the interval of (0, 1)

$\rightarrow$ $\forall\ x \in (1, 2)$ $\Rightarrow x > 0,\ x - 2 < 0,\ x - 1 > 0$

$\Rightarrow x(x-1)(x-2) < 0$

$\Rightarrow 4x(x-1)(x-2) < 0$

$\Rightarrow f'(x) < 0$

$\therefore$ f is strictly increasing function in the interval of (0, 2)

$\rightarrow$ $\forall\ x \in (2, \infty)$ $\Rightarrow x > 0,\ x - 2 > 0,\ x - 1 > 0$

$\Rightarrow x(x-1)(x-2) > 0$

$\Rightarrow 4x(x-1)(x-2) > 0$

$\Rightarrow f'(x) > 0$

$\therefore$ f is strictly increasing function in the interval of $(2, \infty)$

18.

Since, $\vec{a}+\vec{b}+\vec{c}=\vec{0}$ we have,

$\vec{a}\cdot(\vec{a}+\vec{b}+\vec{c})=0$

or $\vec{a}\cdot\vec{a}+\vec{a}\cdot\vec{b}+\vec{a}\cdot\vec{c}=0$

Therefore, $\vec{a}\cdot\vec{b}+\vec{a}\cdot\vec{c}=-|\vec{a}|^2=-1$ ..... (1)

again $\vec{b}\cdot(\vec{a}+\vec{b}+\vec{c})=0$

or $\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}=-|\vec{b}|^2=-16$ ..... (2)

Similarly, $\vec{a}\cdot\vec{c}+\vec{b}\cdot\vec{c}=-4$ ..... (3)

Adding (1), (2) and (3), we have

$2(\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{a}\cdot\vec{c})=-21$

or $2\mu = -21$, i.e., $\mu=\dfrac{-21}{2}$

19.

$L_1 : \vec{r} = (\hat{i} - 2\hat{j} + 3\hat{k}) + t(-\hat{i} + \hat{j} - 2\hat{k})$

$\vec{a_1} = \hat{i} - 2\hat{j} + 3\hat{k}$;

and $\vec{b_1} = -\hat{i} + \hat{j} - 2\hat{k}$

$L_2 : \vec{r} = (\hat{i} - \hat{j} - \hat{k}) + s(\hat{i} + 2\hat{j} - 2\hat{k})$

$\vec{a_2} = \hat{i} - \hat{j} - \hat{k}$;

and $\vec{b_2} = \hat{i} + 2\hat{j} - 2\hat{k}$

Now, $\vec{b_1} \times \vec{b_2}$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 1 & -2 \\ 1 & 2 & -2 \end{vmatrix}$$

$= 2\hat{i} - 4\hat{j} - 3\hat{k}$

$\neq \vec{0}$

$\therefore$ Lines are intersecting lines or skew lines,

$|\vec{b_1} \times \vec{b_2}| = \sqrt{(2)^2+16+9}$

$= \sqrt{29}$

$\vec{a_2} - \vec{a_1} = (\hat{i} - \hat{j} - \hat{k}) - (\hat{i} - 2\hat{j} + 3\hat{k})$

$= 0\hat{i} + \hat{j} - 4\hat{k}$

Now, $(\vec{a_2} - \vec{a_1}) \cdot (\vec{b_1} \times \vec{b_2})$

$= (0\hat{i} + \hat{j} - 4\hat{k}) \cdot (2\hat{i} - 4\hat{j} - 3\hat{k})$

$= 0 - 4 + 12$

$= 8 \neq 0$

Lines are skew lines.

Shortest distance between two skew lines,

$$= \frac{\left|(\vec{a_2}-\vec{a_1})\cdot(\vec{b_1}\times\vec{b_2})\right|}{\left|\vec{b_1}\times\vec{b_2}\right|}$$

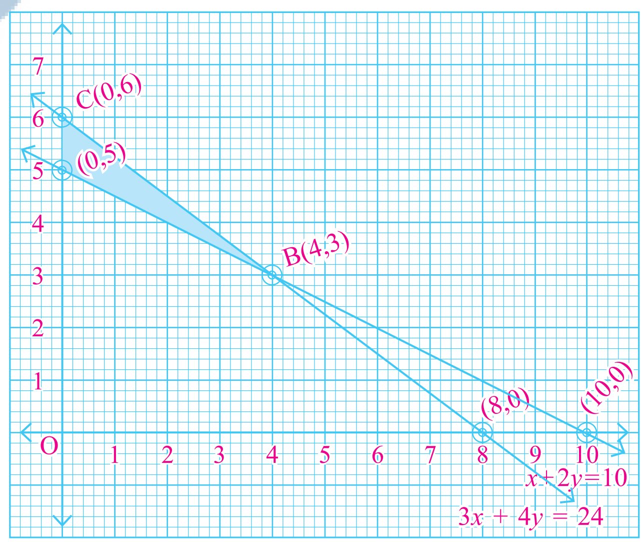
$= \dfrac{8}{\sqrt{29}}$ unit

20.

The shaded region in Fig. is the feasible region ABC determined by the system of constraints (2) to (4), which is bounded.

The coordinates of corner points A, B and C are (0,5), (4,3) and (0,6) respectively.

Now we evaluate Z = 200x + 500y at these points.



Corner Point	Corresponding value of Z
(0, 5)	2500
(4, 3)	2300 $\rightarrow$ Minimum
(0, 50)	3000

Hence, minimum value of Z is 2300 attained at the point (4, 3).

21.

Two fair dice are thrown,

$\therefore$ Sample space

$= \{(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 1), (2, 2), ... (6, 6)\}$

$\therefore n(S) = 36$

Event A : we obtain 6 on 1st die

$\{(6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)\}$

$\therefore \quad n(A) = 6$

Event B : we obtain 2 on 2nd die

$\{(1, 2), (2, 2), (3, 2), (4, 2), (5, 2), (6, 2)\}$

$\therefore \quad n(B) = 6$

$\therefore A \cap B = \{(6, 2)\}$

$\therefore n(A \cap B) = 1$

$$P(A) = \frac{n(A)}{n(S)} = \frac{6}{36} = \frac{1}{6}$$

$$P(B) = \frac{n(B)}{n(S)} = \frac{6}{36} = \frac{1}{6}$$

$$\therefore P(A) \cdot P(B) = \frac{1}{6} \times \frac{1}{6} = \frac{1}{36}$$

$$P(A \cap B) = \frac{n(A \cap B)}{n(S)} = \frac{1}{36}$$

$\therefore P(A) \cdot P(B) = P(A \cap B)$

$\therefore$ Event A and Event B are independent events.

SECTION C

22.

$A^2 = A . A$

$$= \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix} \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 4+0+1 & 0+0-1 & 2+0+0 \\ 4+2+3 & 0+1-3 & 2+3+0 \\ 2-2+0 & 0-1+0 & 1-3+0 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 5 & -1 & 2 \\ 9 & -2 & 5 \\ 0 & -1 & -2 \end{bmatrix}$$

Now, $A^2 - 5A + 6I$

$$= \begin{bmatrix} 5 & -1 & 2 \\ 9 & -2 & 5 \\ 0 & -1 & -2 \end{bmatrix} - 5\begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix} + 6\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & -1 & 2 \\ 9 & -2 & 5 \\ 0 & -1 & -2 \end{bmatrix} + \begin{bmatrix} -10 & 0 & -5 \\ -10 & -5 & -15 \\ -5 & +5 & 0 \end{bmatrix} + \begin{bmatrix} 6 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 6 \end{bmatrix}$$

$$= \begin{bmatrix} 5-10+6 & -1+0+0 & 2-5+0 \\ 9-10+0 & -2-5+6 & 5-15+0 \\ 0-5+0 & -1+5+0 & -2+0+6 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -1 & -3 \\ -1 & -1 & -10 \\ -5 & 4 & 4 \end{bmatrix}$$

23. $A = \begin{bmatrix} 1 & -1 & 2 \\ 3 & 0 & -2 \\ 1 & 0 & 3 \end{bmatrix}$

Co-factor of element 1 $\quad A_{11} = (-1)^2 \begin{vmatrix} 0 & -2 \\ 0 & 3 \end{vmatrix}$

$= (1)(0 - 0)$

$= 0$

Co-factor of element –1 $\quad A_{12} = (-1)^3 \begin{vmatrix} 3 & -2 \\ 1 & 3 \end{vmatrix}$

$= (-1)(9 + 2)$

$= -11$

Co-factor of element 2 $\quad A_{13} = (-1)^4 \begin{vmatrix} 3 & 0 \\ 1 & 0 \end{vmatrix}$

$= (1)(0 - 0)$

$= 0$

Co-factor of element 3 $\quad A_{21} = (-1)^3 \begin{vmatrix} -1 & 2 \\ 0 & 3 \end{vmatrix}$

$= (-1)(-3 - 0)$

$= 3$

Co-factor of element 0 $\quad A_{22} = (-1)^4 \begin{vmatrix} 1 & 2 \\ 1 & 3 \end{vmatrix}$

$= 1(3 - 2)$

$= 1$

Co-factor of element –2 $\quad A_{23} = (-1)^5 \begin{vmatrix} 1 & -1 \\ 1 & 0 \end{vmatrix}$

$= -1(0 + 1)$

$= -1$

Co-factor of element 1 $\quad A_{31} = (-1)^4 \begin{vmatrix} -1 & 2 \\ 0 & -2 \end{vmatrix}$

$= 1(2 - 0)$

$= 2$

Co-factor of element 0 $\quad A_{32} = (-1)^5 \begin{vmatrix} 1 & 2 \\ 3 & -2 \end{vmatrix}$

$= -1(-2 - 6)$

$= 8$

Co-factor of element 3 $\quad A_{33} = (-1)^6 \begin{vmatrix} 1 & -1 \\ 3 & 0 \end{vmatrix}$

$= 1(0 + 3)$

$= 3$

$$adj\ A = \begin{bmatrix} 0 & 3 & 2 \\ -11 & 1 & 8 \\ 0 & -1 & 3 \end{bmatrix}$$

$$A(adj\ A) = \begin{bmatrix} 1 & -1 & 2 \\ 3 & 0 & -2 \\ 1 & 0 & 3 \end{bmatrix} \begin{bmatrix} 0 & 3 & 2 \\ -11 & 1 & 8 \\ 0 & -1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 0+11+0 & 3-1-2 & 2-8+6 \\ 0+0+0 & 9+0+2 & 6+0-6 \\ 0+0+0 & 3+0-3 & 2+0+9 \end{bmatrix}$$

$$A(adj\ A) = \begin{bmatrix} 11 & 0 & 0 \\ 0 & 11 & 0 \\ 0 & 0 & 11 \end{bmatrix} \quad .... (1)$$

$$(adj\ A)\ A = \begin{bmatrix} 0 & 3 & 2 \\ -11 & 1 & 8 \\ 0 & -1 & 3 \end{bmatrix} \begin{bmatrix} 1 & -1 & 2 \\ 3 & 0 & -2 \\ 1 & 0 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 0+9+2 & 0+0+0 & 0-6+6 \\ -11+3+8 & 11+0+0 & -22-2+24 \\ 0-3+3 & 0+0+0 & 0+2+9 \end{bmatrix}$$

$$(adj\ A)\ A = \begin{bmatrix} 11 & 0 & 0 \\ 0 & 11 & 0 \\ 0 & 0 & 11 \end{bmatrix} \quad .... (2)$$

$$|A| = \begin{vmatrix} 1 & -1 & 2 \\ 3 & 0 & -2 \\ 1 & 0 & 3 \end{vmatrix}$$

$$= 1(0 - 0) + 1\ (9 + 2) + 2\ (0 - 0)$$

$$= 11$$

$$|A|\ I_3 = 11 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 11 & 0 & 0 \\ 0 & 11 & 0 \\ 0 & 0 & 11 \end{bmatrix} \quad .... (3)$$

From equation (1), (2) and (3),

$A(adjA) = (adjA)\ A = |A|\ I$

24.

$y = 500e^{7x} + 600e^{-7x}$

Diffeentiate w.r.t. x,

$$\frac{dy}{dx} = 500\ e^{7x}\ (7) + 600\ e^{-7x}\ (-7)$$

Now, differentiate again w.r.t. x,

$$\therefore \frac{d^2y}{dx^2} = 500\ e^{7x}\ (7)(7) + 600\ e^{-7x}\ (-7)(-7)$$

$$\therefore \frac{d^2y}{dx^2} = 49\ (500\ e^{7x} + 600\ e^{-7x})$$

$$\therefore \frac{d^2y}{dx^2} = 49y$$

25.

We have

$$f(x) = 3x^4 + 4x^3 - 12x^2 + 12$$

or $f'(x) = 12x^3 + 12x^2 - 24x$

$= 12x\ (x - 1)(x + 2)$

or $f'(x) = 0$ at $x = 0,\ x = 1$ and $x = -2$.

Now, $f''(x) = 36x^2 + 24x - 24$

$= 12\ (3x^2 + 2x - 2)$

or $\begin{cases} f''(0) = -24 < 0 \\ f''(1) = 36 > 0 \\ f''(-2) = 72 > 0 \end{cases}$

Therefore, by second derivative test, $x = 0$ is a point of local maxima and local maximum value of f at $x = 0$ is $f(0) = 12$ while $x = 1$ and $x = -2$ are the points of local minima and local minimum values of f at $x = -1$ and -2 are $f(1) = 7$ and $f(-2) = -20$, respectively.

26.

$$I = \int \frac{5x\ dx}{(x+1)(x^2+9)}$$

$$\frac{5x}{(x+1)(x^2+9)} = \frac{A}{x+1} + \frac{Bx+C}{x^2+9}$$

$\therefore\ 5x = A(x^2 + 9) + (Bx + C)(x + 1)$

→ Now, Take $x = -1$,

$\therefore\ -5 = A(10) + (Bx + C)(0)$

$\therefore\ A = -\frac{1}{2}$

→ Now, Take $x = 0$,

$\therefore\ 0 = 9A + B(0) + C(1)$

$\therefore\ 0 = \frac{-9}{2} + 0 + C$

$\therefore\ C = \frac{9}{2}$

→ Now, Take $x = 1$,

$\therefore\ 5 = 10A + (B + C)(2)$

$\therefore\ 5 = 10A + 2B + 2C$

$\therefore\ 5 = \frac{-10}{2} + 2B + 2\left(\frac{9}{2}\right)$

$\therefore\ 5 = -5 + 2B + 9$

$\therefore\ 2B = 1$

$\therefore\ B = \frac{1}{2}$

$$I = \int \frac{5x\ dx}{(x+1)(x^2+9)}$$

$$= \frac{-1}{2}\int \frac{dx}{x+1} + \int \frac{\frac{1}{2}x + \frac{9}{2}}{x^2+9}\ dx$$

$$= \frac{-1}{2}\int \frac{dx}{x+1} + \frac{1}{2(2)}\int \frac{2x}{x^2+9}\,dx + \frac{9}{2}\int \frac{dx}{x^2+9}$$

$$I = \frac{-1}{2}\int \frac{dx}{x+1} + \frac{1}{4}\int \frac{\frac{d}{dx}(x^2+9)}{x^2+9}\,dx + \frac{9}{2}\int \frac{dx}{(x)^2+(3)^2}$$

$$= \frac{-1}{2}\log|x+1| + \frac{1}{4}\log(x^2+9) + \frac{9}{2}\left(\frac{1}{3}\right)\tan^{-1}\left(\frac{x}{3}\right) + c$$

$$I = \frac{-1}{2}\log|x+1| + \frac{1}{4}\log(x^2+9) + \frac{3}{2}\tan^{-1}\left(\frac{x}{3}\right) + c$$

27.

➪ $\frac{dy}{dx} + y\cot x = 4x\,\text{cosec}\,x$ ... (1)

Comparing equation (1) with $\frac{dy}{dx} + P(x)\,y = Q(x)$

$P(x) = \cot x$

$Q(x) = 4x\,\text{cosec}\,x$

→ Integrating factor I.F. $= e^{\int P(x)\,dx}$

$= e^{\int \cot x\,dx}$

$= e^{\log|\sin x|}$

$= \sin x$

→ Multiply equation (1) with $\sin x$,

$\therefore \frac{dy}{dx}\sin x + y\cot x\sin x = 4x\,\text{cosec}\,x\sin x$

$\therefore \frac{d}{dx}(y\sin x) = 4x$

$\therefore y\sin x = \int 4x\,dx$

$\therefore y\sin x = 2x^2 + c$ ... (1)

→ If $x = \frac{\pi}{2}$ and $y = 0$, then

$\therefore 0 = 2\left[\frac{\pi^2}{4}\right] + c$

$\therefore c = -\frac{\pi^2}{2}$

→ Putting the value of c in equation (1),

$\therefore y\sin x = 2x^2 - \frac{\pi^2}{2}$, where, $\sin x \neq 0$

Which is the particular solution of given differential equation.